CHAPTER 2

On the history of forming prime numbers tables and determining the smallest divisor of composite numbers

2.1. Famous tables of prime numbers and divisors of composite numbers

To considering the importance of the recognition of the prime numbers and factorization numbers to them and on the other side, difficulties of recognition of primality and factorization, some tables (of course restricted) of prime numbers and divisors or the smallest divisor of the composite numbers have been prepared. Some of the most famous of these tables are:

Lehmer, D.N; Table of factors for the first ten millions numbers containing the smallest factor of every number not divisible by 2,3,5 and 7 between the limits "1" and "10,017,000" list of prime numbers from "1" to 10,006, 721[1].

Kulik, J. ph. Poletti.L and Porter. R. list of prime numbers of the 11-th million (precisely: from 100006741 to 10999997)[2].

Preparing the prime numbers table and the table of factors have old antecedent. In chapter "5" of the book "Arithmetic" by Fibonacci (1202) a list of prime numbers from "11" to "97" has been given, and also he mentioned the factorization of numbers 12-100 to their factors. Cathaldy provided a table from the prime numbers smaller than 750 in his research about perfect numbers. Then bigger tables had prepared gradually. Some characteristics of some of the most important tables are presented in the next page, except 3 tables that have been mentioned at first[3].

Tables of Divisors

Divisors table	Date	Provider
1-2,856,000	1785	Felkel
1-3,000,000	1817	Burkhard
7,000,000-9,000,000	1863	Dazet
1-100,000,000	1867	Kulik
3,000,000-6,000,000	1883	Glishare

The propounded tables are based on the Burkhard, Kulik, Glishare and Dazet works. Preparing big tables is very tedious and needs to be patient, attempt, tolerance and every body who looks at these tables, could not restraint admiring the people who have done this difficult job with no reward. Biller in his book "Entertainments in number theory" said “By seeing the attempt ion in this disordered world with war and continual conflict, it seems that the person who endowed the big part of their life to this work, must have found the secret of calmness and peace that no wordily greedy person can reach it absolutely". Antonio Felkel, whose name was mentioned in the previous pages, in 1776, computed divisors of all numbers up to 2,000,000. The first part of his table (up to 408000) was printed by the Vian’s royal treasury but did not have costume and in Austria

1. Carnegie Institution of Washington Publication 105, 1909 (165, 1914)
2. From tables of manuscript, Amsterdam 1951
3. Historical subject of this part is based on the first volume of "History of theory of numbers" Dickson, "primary numbers" by Biller. Table in the next page is from the last book.

war with Turkey, was used for preparing cartridge. Felkel was planning to take back the rest of his table, but the treasury refused and since he disappointed to gain it, again he computed the divisors of the numbers from 408000 to 2856000. Philip Kulik's motivation in preparing the factor's tables is inconceivable. He had prepared this table on his own, at the. Lehmer verified the first volume of these 8 volumes in Royal academy in Wiena and described it in the preface of factor's table with enthusiasm and anxiety to impress the readers. The Kulik's table was used in preparing the table of "Kulik, Poletti and Forter" that we have mentioned before, and reformed its mistakes.

2.2. Calculation of tables

According to our previous statements, it is clear that easily but with patience, it is possible to prepare the table numbers.

In fact, it took a long time to determine the prime numbers smaller than 10 million. These tables are printed and for each of the first 10 million numbers, the smallest odd divisor was written in front of them. Therefore, each number that there is not a number written in front of, (has no divisor), it is prime number. Also others have calculated the prime numbers between two big numbers (bigger than 10 million) for example, the prime numbers between 100,000,000 and 100,100,000. It is clear that these calculations are very detailed and long, because all numbers between these two numbers that are multiplicand of prime numbers smaller than approximate square root of 100 million should be omitted.

(It means smaller prime numbers less than 10,000 that are 2000 numbers).

Extending and completing the calculators help to speed up the calculations. However it must be looked after to prevent the mistakes of copying, and printing.

It is expected, that this valuable work helps to improve our knowledge about prim numbers greatly and this analysis (that we can not explain it, in this book).

However we must confess, there are a lot of ambiguous points despite so many attempts to recognize the prime numbers. But with patient researchers who have done researches about the prime numbers tables, it seems that some of these theorems will be confirmed by the mathematician's reasoning. There is no doubt that this experimental and sensuous method has been ursine successfully by some of the mathematicians. And here, it isn't un-useful to present some of the researches to cleat the prime numbers recognition category.

2.3. Stochastic's Theorem

One of the most attractive theorems that are resulted from these statistic researches is stochastic theorem. This theorem has perfect relation with the sieve alternation's rule (that we have mentioned it before), But we can say that random theorem completes this rule in some cases.

It is clear that divisible numbers by prime divisors 2, 3 and 5 are repeated in every 30 successive numbers frequently. The Numbers which are not divisible by 2, 3 and 5 are in one of below form:

30n+1 30n+7 30n+11
30n+13 30n+17 30n+19
30n+23 30n+29

(Or: $3n + k$, $n \in IN$, $k \in \{1,7,11,13,17,19,23,29\}$).

In the first ten numbers, There are two numbers and in the second ten numbers, four and in the third ten numbers, there are two numbers of these.

Among these numbers, there are 26 numbers that are between "1" to "100" and 28 numbers between 101 to 200 and 26 numbers between 201 to 300 and these numbers are repeated successively in every next 100 numbers in this way.

Now, if we consider the prime numbers between 2 to 13, their multiplications are equal or 30030. Here again the same procedure is repeated but succession period is equal to 30030 instead of 30 and also if we devoid successive numbers to the packs containing 30 numbers, (the packs that have 8 un-divisible numbers by 2 and 3 and 5), the number of numbers that are divisible by 7, 11, 13 in these patches will be various. For the first ten packs (1 to 30 and 31 to 60 and … and 271 to 300) the numbering of these numbers are 3,1,1,2,3,2,3,2,3,1,... and in this way the number of numbers that are not divisible by 2,3,5,7,11,13 will be equal to 5,7,7,6,5,6,5,6,7 (that are calculated from subtracting the previous values of 8). By the same way, various values will be calculated for the first 1001 packs of the 30 number namely up to 30030.

Only when we reach this number, the previous values will be repeated prodigally. It means that there are 5 numbers between 30031 to 30060 that they are not divisible by 2,3,5,7,11,13 (that equal the number of these numbers, when we consider the number between 1 to 30).

It seems that when we want to increase the number of prime numbers, the length of succession period will grow rapidly. So if we want to search for the numbers that are not divisible by any of smaller prime numbers than 100, the succession period will be equal multiplication of these numbers that is a number with 30 digits approximately, and it is clear that the periodic method can not be used neither for the numbers that their primality can be investigated nor for the numbers bigger than one million that their smallest prime factor is near to 1000.

Now we can explain the random theorem: it concerns the distribution of the prime numbers in the consecutive packs of numbers. For example the distribution in the packs of 100- numbers, follows the random theorem, with considering the succession period for the smaller prime numbers. This will decrease the numbering of the prime numbers which exist probably.

If we limit the factors to 2, 3, 5 there will be 26 or 28 numbers in every 100 successive numbers, so that they are not divisible by 2, 3, 5, and if we consider the factor 7 these numbers will even decrease. On the other side it is possible to determine the average of the prime numbers which exist between every 100 numbers, for example the numbers between 1,000,000 and 1,100,000 and these numbers will be more than 7.

If the distributing of prime numbers were requited, there would be 7 prime numbers in every 100 number. But in fact although the number 7 exists frequently, the real number of prime numbers is often different from 7 in 100-number intervals and varies from 0 to 14.

Comparing the prime numbers to the obtained resorts, clarifies that in measure 100, in a box, there are 7 black balls in every 100 balls. It means 7 black balls and 93 white balls. It’s clear that difference between the numbers of prime numbers to their average is less than the ratio that is shown here. Therefore, if is necessary to pay attention to certain recurrence that is verified for small prime numbers.

The recurrence arranged this phenomenal. If we consider only prime factors 2, 3, 5, we will see that the number of numbers that are prime probably, (It means the numbers that are not divisible by 2, 3, 5) is equal to 26 or 28 in every recurrence 100 numbers. Therefore, if is not necessary to consider a box with 100 balls. But we must consider a box for example which has 20 white balls against every 7 black balls (in measure 27 balls). It is possible to consider more prime numbers.

The multiplication of prime number up to 17 is equal to 510510 and up to 19 is equal to 9699690. The numbers which are under our consideration are between these two multiplications and if we consider prime numbers up to 17, another arrangement will be established. There are $2\times 4\times 6\times 10\times 12\times 16$ numbers between the first 510510 numbers that are not divisible by prime number up to 17.

It means that there are 5760×16 numbers between 500000 numbers and it will be %18 of them. Now we consider the balls in every 18 balls, 7 black balls and 11 white balls and it is verified that the ratio calculated by theoretical hypothesis is very much closer to the observed conditions.

Now we can arrange random theorem:

We consider the pack "T" which is randomly chosen from the successive 100 numbers for example between 2,000,000 and 2,100,000. The probability that the pack "T" includes "p" prime numbers is equal to the probability of existing "p" black balls in "q" balls in a box that has "n" black balls and "$q-n$" white balls. The numbers "q" and "n" are selected in this way: the number "q" is the possible number of number which can be in every 100 prime numbers, in clouding the numbers that are divisible by small prime numbers (up to 17), and the number "n" is the average number of prime numbers which are in every 100 number of studied pack.

It is clear that it is not correct to speak about the primality of a supposed number, because if a number is determined, it is easy to determine that it is prime or not. But we can speak about primality of a number if it is selected randomly with some condition. For example suppose that the number of a lottery winner is a prime number.

Here we use classification pattern to shorten numerical tables.

The following table is designed for the interval (31,211]:
Note: The Composite numbers are bold.

$(30,p)=1$	+30	+60	+90	+120	+150	+180
7	37	67	97	127	157	**187**
11	41	71	101	131	**161**	191
13	43	73	103	**133**	163	193
17	47	**77**	107	137	167	197
19	**49**	79	109	139	**169**	199

×	7	11	13
7	49		
11	77	121	
13	91	143	169
17	119	187	
19	133	209	

23	53	83	113	**143**	173	**203**
29	59	89	**119**	149	179	**209**
31	61	**91**	**121**	151	181	211

23	161		
29	203		

• Multiplication table determining composite numbers in the interval (31,211]

2.3.1. Note. To eliminate composite number from the table it suffices to form the multiplication table of the interval under consideration.

Considering the multiplication table for the aforementioned interval 12 these composite numbers are obtained which are to be eliminated from the table associated with the cycle 30*s*+*p*.

For the interval (211, 2311] the table of classes is as the following:

(210,P) =1	+210	+420	+630	+840	+1050	+1260	+1470	+1680	+1890	+2100
11	**221**	431	641	**851**	1061	**1271**	1481	**1691**	1901	2111
13	223	433	643	853	1063	**1273**	1483	1693	**1903**	2113
17	227	**437**	647	857	**1067**	1277	1487	1697	1907	**2117**
19	229	439	**649**	859	1069	1279	1489	1699	**1909**	**2119**
23	233	443	653	863	**1073**	1283	1493	**1703**	1913	**2123**
29	239	449	659	**869**	**1079**	1289	1499	1709	**1919**	2129
31	241	**451**	661	**871**	**1081**	1291	**1501**	**1711**	**1921**	2131
37	**247**	457	**667**	877	1087	1297	**1507**	**1717**	**1927**	2137
41	251	461	**671**	881	1091	1301	1511	1721	1931	2141
43	**253**	463	673	883	1093	1303	**1513**	1723	1933	2143
47	257	467	677	887	1097	1307	**1517**	**1727**	**1937**	**2147**
53	263	**473**	683	**893**	1103	**1313**	1523	1733	**1943**	2153
59	269	479	**689**	**899**	1109	1319	**1529**	**1739**	1949	**2159**
⋮	...	...	...	...	...	...	...	...	...	...
191	401	**611**	821	1031	**1241**	1451	**1661**	1871	2081	**2291**
193	**403**	613	823	**1033**	**1243**	**1453**	1663	1873	2083	2293
197	**407**	617	827	**1037**	**1247**	**1457**	1667	1877	2087	2297
199	409	619	829	1039	1249	1459	1669	1879	2089	**2299**
209	419	629	839	1049	1259	**1469**	**1679**	1889	**2099**	2309
211	421	631	**841**	1051	**1261**	1471	**1681**	**1891**	**2101**	2311

According to the tables, just mentioned, it follows that prime numbers are generated by the following general formula:

$$(p, S_{p_o} k_{p_1}) = 1 \ : \ P(k_{p_1}) = S_{p_o} k_{p_1} + p \qquad (1)$$

In the above table S_{p_o} is called the cycle constant and we consider it to be equal to one of the numbers in the following sequence:

$$S_{p_o} : \underbrace{2.3}_{6}, \underbrace{2.3.5}_{30}, \underbrace{2.3.5.7}_{210}, \underbrace{2.3.5.7.11}_{2310}, \underbrace{2.3.....13}_{30030}, ..., \underbrace{2.3.5.....p_o}_{S_{p_o}}$$

In (1) "p" is called the generating number. This definition justifies adding a multiple of S_{p_o} to "p" would result in new prime numbers. The positive integer k_{p_1} depends on the cycle constant (S_{p_0}); i.e., if we consider the cycle constant as follows (p_1 is the prime number following p_0)

$$S_{p_o} = 2.3.5.7.11.13.17..... p_0$$

The upper and lower bounds for k_{p_1} are as follows:

$$(k_{p_1} \in IN) 1 \le k_{p_1} < p_1$$

We conclude that the set of prime numbers consists of a union of numerical cycles as described in (1). Each of these cycles is consisted of "n" numerical classes. The number "n" is in fact equal to the ratio of the distance of two consecutive cycles constant to the first one:

$$n = \frac{2.3.5..... p_0.p_1 - 2.3.5..... p_0}{2.3.5.... p_0} = p_1 - 1 \quad \text{(The number of classes)}$$

According to this prime numbers could be regularized meaning that prime numbers associated with symbols "$k_{p_1} =$" or "$k_{p_1} \ne$" could be listed in a table with formulas of the form (1). For example, according to the former tables prime numbers of the cycle $S_3 = 6$, $S_5 = 30$ for $p = 7$ and $p = 11$ could be written as follows $(1 \le k_{p_1} < p_1)$:

$$k_5 \ne 3 \ : \ p(k_5) = 6k_5 + 7$$
$$k_7 \ne 6 \ : \ p(k_7) = 30k_7 + 7$$
$$k_7 \ne 5 \ : \ p(k_7) = 30k_7 + 11$$

2.3.2. Note. If in case the number of prime numbers of classes in a cycle is larger, it's better to use the symbol "$k_{p_1} \ne$", otherwise, the symbol "$k_{p_1} =$" is used.

As another example the cycle $s_5 = 30$, $s_7 = 210$, $s_{11} = 2310$ for $p = 13$ are as follows:

$$k_7 \ne 4 \ : \ p(k_7) = 30k_7 + 13$$
$$k_{11} \ne 6,9 \ : \ p(k_{11}) = 210k_{11} + 13$$
$$k_{13} \ne 6,7,8,11,12 \ : \ p(k_{13}) = 2310k_{13} + 13$$

Here, the following table is used to regularize prime number. This table could be extended to any given arbitrary cycle.

Prime numbers table in terms of consecutive cycles $(p(k_{p_1}) = s_{p_o} k_{p_1} + p)$.

Table of prime numbers (H.M)

s_{p_o}, k_{p_1} / p	$s_3 = 6$ k_5	$s_5 = 30$ k_7	$s_7 = 210$ k_{11}	$s_{11} = 2310$ k_{13}	$s_{13} = 30030$ k_{17}	$s_{17} = 510510$ k_{19}
5	< 5					
7	≠ 3	≠ 6				
11		≠ 5	≠ 1,4,6,8			
13		≠ 4	≠ 6,9	= 6,7,8,11,12		
17		≠ 2	≠ 2,5,10	≠ 1,5,8,11	= 1,2,3,8,9,10,11,13	
19		≠ 1,5	≠ 3,9,10	= 2,3,6,7,9,12	= 5,7,8,10,14,16	= 1,3,4,9,11,12,16,18
23		≠ 4,6	≠ 5,8,10	= 1,2,6,7,8,12	= 1,7,8,10,13,15,16	= 2,6,10,17
29		≠ 3,6	≠ 4,5,9	= 1,2,3,5,11,12	= 1,2,9,11,13,15,16	= 7,8,9,11,12,14,15,17
31		≠ 2,3	= 1,3,6,10	= 1,2,3,10,12	≠ 1,4,5,9,11,14,16	= 3,5,9,11,13,17,18
37			= 2,4,5,6,10	≠ 6,7,9,10,12	= 3,4,7,9,14,15,16	= 3,4,5,6,7,13,15,17
41			≠ 3	= 1,3,4,6,8	= 1,2,6,9,16	= 1,10,11,12,15,18
43			≠ 1,7	≠ 1,3,7,9	= 2,4,5,8,10,13,15	= 1,4,7,9,10,11,14,17
47			≠ 7,8,9,10	≠ 2,4,8,9,10	≠ 1,3,6,9,11,15	= 2,5,16,18
53			≠ 2,4,6,9	≠ 1,5,8,9,10	= 5,6,7,9,11,15,16	= 2,5,7,14
59			= 1,2,5,6,9	≠ 1,3,4,5,6	= 1,3,5,6,9,11,13,14	= 1,7,8,9,11,13,14,15,18
61			≠ 2,4,5	≠ 1,3,7,8,11	= 1,4,5,6,12,14,16	= 2,3,5,6,8,10,12
67			≠ 3,7,9,10	= 1,3,5,9,10	= 1,2,5,6,7,9,10	= 2,4,7,8,9,12,15,18

s_{p_o}, k_{p_1} / p	$s_7 = 210$ k_{11}	$s_{11} = 2310$ k_{13}	$s_{13} = 30030$ k_{17}	$s_{17} = 510510$ k_{19}
71	= 1,2,3,4	≠ 7,8,9,10,11	= 5	≠ 3,6,7,8,11,14,16,18
73	= 1,5,7,8	= 1,6,8,10,12	≠ 6,8,10,11,12,14,15	= 1,2,5,6,9,11,15
79	≠ 1,6,9	= 1,4,7,12	≠ 3,5,7,9,10,13,16	= 1,5,6,10,11,13,14,17
83	= 1,2,7,9	≠ 6,8,10,11	= 1,3,6,11,14,15,16	= 4,8,10,12,14,17,18
89	= 2,3,4,7,9	= 1,3,9,10,12	= 1,2,4,5,7,13,16	= 3,4,6,8,11,12,17,18
97	≠ 2,5,6,10	= 3,4,7,9,10,12	= 3,5,10,11,12,13	= 3,5,6,8,14,16
101	≠ 3,8,9,10	= 1,2,4,10	≠ 1,4,5,7,10,15,16	= 1,3,5,9,15
103	≠ 4,6,7	≠ 1,3,5,9,11	= 1,4,11,13,16	= 1,2,3,5,7,10,11,17
107	≠ 2,3,5,7	= 1,5,6,8,9,12	= 1,4,11,13,16	= 1,2,5,6,11,14,17
109	= 3,7,8,9	= 2,3,4,9,10	≠ 4,5,10,12,14,15,16	= 1,2,4,5,6,7,16,17
113	≠ 1,2,3,8	= 1,2,3,8,9,11	= 3,4,7,8,10,13,15	= 4,11,17
121	≠ 4,7	F	F	F
127	≠ 5,6,8,10	= 1,3,5,10,11,12	= 3,4,6,10	= 3,5,6,7,8,11,15,16
131	≠ 1,2,6,10	≠ 3,6,8,10	= 1,6,8,10,14,15	= 3,4,8,9,10,11,14
137	≠ 3,6,8	= 1,4,6,8	≠ 1,2,4,8,10,11,16	= 2,5,6,8,10,11,13,14
139	≠ 2,4,5,8	= 2,3,5,6,9	= 1,8,10,11	= 2,3,4,7,13,15,16
143	≠ 6,9	F	F	F

s_{p_o}, k_{p_1} / p	$s_7 = 210$ k_{11}	$s_{11} = 2310$ k_{13}	$s_{13} = 30030$ k_{17}	$s_{17} = 510510$ k_{19}
149	$= 1,2,6,7,9$	$= 1,3,5,6,7,9$	$= 2,3,5,7,12,13,14,15$	$= 4,8,9,13,14,18$
151	$\neq 1,3,6,9$	$= 4,5,6,10,11$	$= 1,5,6,7,9,12,14,15$	$= 3,8,12,13,17$
157	$= 1,2,3,4,7$	$= 1,4,8,9$	$= 1,2,3,4,6$	$= 6,7,12,14,15$
163	$= 1,5,6,9$	$= 1,2,4,7,12$	$= 2,4,10,13$	$= 2,4,5,7,9,14,15,17,18$
167	$\neq 1,4,9$	$= 1,2,5,11$	$= 1,6,9,15,16$	$= 1,3,4,5,8,10,11,13$
169	$= 1,4,6,10$	$\neq 1,3,4,8,12$	F	F
173	$\neq 3,7,8$	$\neq 1,5,7,8,10$	$= 1,3,4,5,9,14$	$= 1,4,9,12,13,16,18$
179	$\neq 7,8,10$	$= 2,3,4,7,10,11$	$= 4,5,11,14$	$= 2,3,5,7,9,12,13$
181	$\neq 1,6,7,9$	$= 2,4,5,8,12$	$\neq 2,4,5,11,13,14,15$	$= 1,4,7,8,11,14,15,16,17$
187	$\neq 3,4,9$	F	F	F
191	$\neq 2,5,7,10$	$\neq 1,2,5,12$	$\neq 1,4,5,8,11,14,16$	$= 3,7,9,12,15$
193	$\neq 1,5$	$\neq 3,6,8,12$	$= 1,5,7,8,9,10,14,15$	$= 4,6,9,11,14,17$
197	$\neq 1,4,5,6$	$= 2,3,4,6,10,12$	$= 2,7,8,10$	$= 1,2,4,6,12,13,16,18$
199	$\neq 10$	$= 3,4,7,8,11,12$	$= 2,3,4,6,10,15$	$= 1,3,7,8,10,14,15,16,18$
209	$\neq 2,6,7$	F	F	F
211	$= 1,2,4,6,10$	$\neq 3,4,5,12$	$= 1,2,4,6,7,10$	$= 5,9,14,15,16,18$
221		$\neq 2,5,7,11$	F	F
223		$= 4,6,9,11,12$	$= 1,3,5,9$	$= 2,4,5,8,11,17$
227		$= 4,5,6,9,10,12$	$= 5,7,11,15,16$	$= 5,6,8,9,10,12,15$
229		$= 1,3,5,9,11$	$= 1,2,4,5,12$	$= 5,6,7,9,17$
233		$\neq 2,3,6,7$	$= 2,5,6,8,11,12,16$	$= 2,4,7,8,9,13,17,18$
...	...	...	...	...

2.3.3. Note This table gives any prime less than $s_{19} = 2 \times 3 \times ... \times 19$, i.e. $s_{19} = 9699690$.

The following table (I) is designed for the numbers that are divisible by prime numbers of classes:

a: "2" , "3" and "5". b: from "7" up to "19".
c: from "23" up to "97". d: greater than "100".
e: prime numbers.

Table (I)

N	*A*	*b*	*c*	*d*	*e*	Error controlling
2000000+ 0001000	7334	955	497	509	705	
10001000	7334	959	508	508	691	
20001000	7332	956	502	517	693	
30001000	7334	954	513	509	691	(+1)
40001000	7334	956	505	534	670	(-1)
50001000	7332	957	512	503	696	

N	*A*	*b*	*c*	*d*	*e*	Error controlling
60001000	7334	957	504	511	684	
70001000	7334	954	507	531	674	
80001000	7332	960	507	515	686	
90001000	7334	959	502	531	674	
summand	73334	9567	5057	5168	6874	

For every column of this table, a table with more details is prepared that we show that for column e, namely the column of prime numbers, (table (II) in next page). Every row of these tables justifies a pack of 10000 numbers like every row of the previous table.

Table (II)

	1	2	3	4	5	6	7	8	9	10	11	12	13
0001000			3	7	13	25	10	13	16	10	2		1
10001000			3	8	10	20	22	17	15	2	1	2	
20001000		2	5	9	9	13	23	15	11	8	5		
30001000		2	2	4	14	17	25	22	4	6	3		1
40001000	2	1	2	8	13	26	14	11	11	11		1	
50001000	1		3	5	10	23	23	13	12	7	1	2	
60001000			4	6	12	25	18	12	11	7	3	2	
70001000		2	5	7	13	14	23	16	14	3	3		
80001000		2	2	9	9	17	25	18	11	3	3	1	
90001000		2	3	5	15	17	22	20	10	6			
summand	3	11	32	68	118	197	205	157	115	63	21	8	2

And this pack is divided into 100 packs of 100 numbers. The recorded numbers in various columns of the first row of this table, determine the number of prime numbers in packs of 100 numbers from these 100 packs (that started from 2000001). So there are three packs of these packs that each one includes 3 prime numbers. 7 packs include 4 prime numbers, 13 packs include 5 prime numbers. 25 packs include 6 prime numbers, 10 packs include 7 prime numbers, 13 packs include 8 prime numbers, 16 packs include 9 prime numbers, 10 packs include 10 prime numbers, 2 packs include 10 prime numbers, 2 packs include 11 prime numbers and at last one pack includes 13 prime numbers and the next rows follow this order.

Investigating this table, shows that some disorders exist in every row, about distribution of 100 numbers which include the number of prime numbers (from 1 to up 13), (the average number that obtained from table (I) is 6.874).

The first row has two maximum numbers (25 and 16).

The numbers of six-th column decrease unexpectedly for the rows 20001 and 70001 (13 and 14) and also in rows 00001 (10) and 40001 (14). But the seams show a kind of ordered distribution of prime numbers according to the first law of lap lass – gauss, with an important distribution when the number of prime numbers equals "1" in some packs of 100 numbers and in some other equals 13. But this distribution is the result of the random that obtained for 1000 series, and probability of existing 17 prime numbers is in every series. Like a box which includes 17000 balls that 6874 balls are black. The observed disorders are usual and it confirms the random theorem.

Now we search the obtained results from previous table, about 10000 packs of 100 numbers up to 10 million, namely numbers between 9000001 up to 10 million.

Table (III)

$\times 10^5$	n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
90 to 91	6182		4	21	54	124	176	186	179	131	82	32	9	2		
91 to 92	6245		2	20	60	126	151	195	189	127	78	29	15	6	2	
92 to 93	6259		2	26	48	98	161	228	190	127	77	24	12	6	1	
93 to 94	6223		3	21	53	103	161	215	195	114	73	24	12	3	2	1
94 to 95	6177		6	24	56	105	189	200	166	120	84	41	6	3		
95 to 96	6271	1	1	13	54	124	157	194	206	139	59	32	10	8	2	
96 to 97	6202		3	25	55	111	176	212	170	112	83	34	14	4	1	
97 to 98	6208	1	3	17	60	122	173	197	167	133	68	40	17	2		
98 to 99	6181		1	15	67	113	180	208	178	119	70	28	15	3	2	1
99 to100	6134		4	21	65	104	188	209	178	115	67	33	10	5		1
summand	62082	2	29	203	572	1140	1712	2044	1818	1237	751	317	120	42	10	3

In the first of table (III) which is identified by 10^5, there are 100,000 units in every row, it means 1000 packs of 100 numbers, for example from 90×10^5 up to 91×10^5, then in column "*n*" the number of prime numbers of this interval is written, and in the next columns that numbers from "1" up to "14" are written above them, the number of 100-number packs is identified which has "1" or "2",..., or "14" prime numbers. As it is found out from the sums, it seems that the distribution of prime numbers are arranged completely and almost, there is a result for 10 million like the same result for 3 million. Therefore, we can accept that a general rule exists.

It is surprising to use this statistics of the prime numbers about approximate distances, very big number like 100 million. The column is very interesting because it shows that how the frequency of prime numbers is near the indeterminate number "*N*". This column results that there is an important diversity for big distances in relation to this theoretical frequency.

For distances of 100,000, the number of prime numbers that are in 10^{th} million is equal to:

6182 6245 6223 6177
6271 6202 6201 6134

It seems that the first number is very small abnormally, but its second root does not have big diversity in relation to average value so that it is possible to consider it as normal.

If we put 100,000 instead of 200,000, disordering will decrease so that if we add adjacent numbers two by two, will have:

12427, 12482, 12448, 12410, 12315

and with addition of three-to-three of these numbers (distances of 300,000):

18686 18671 18591 (61314)

The obtained numbers will approach to each other continuously. Finally by adding 4-to-4 or 5-to-5:

24909	24858
31086	30996

2.4. Another research on stochastic theorem

Mr. Giuseppe Palama had presented and it led to search about random theorem. The subject is the definition of prime numbers in the form $1848x^2 + y^2$ and between 11000000 and 11100000.

Euler had shown that if integer number "n" chooses values, only numbers in $nx^2 + y^2$ form will be prime numbers. He had determined a lot of these values of "n" that the greatest of them is $1848 = 42 \times 44 = 43^2 - 1$. This remark of Euler proves (Mr. Palama's statistics), when he determined 23 prime numbers in $1848\,x^2 + y^2$ between the numbers 11×10^6 and $11 \times 10^6 + 10^5$.

Also in every 100 packs of 1000 numbers there are in average 2.03 prime numbers with above form in distance 100000. We can determine easily number "N" that is indicator of the number of packs of 1000 numbers and include 0, 1, 2, 3, 4, 5, … prime numbers, so the following table obtains:

Table (IV)

	0	1	2	3	4	5	6	7	8	summand
N…	7	25	38	23	3	4	0	0	0	100
Poisson	13.5	27.1	27.1	18	9	3.6	1.2	0.3	0.1	99.9

Also he mentioned a form of non – prime odd numbers in this distance but this form is not complete.

In the third row, we have recorded the theoretical values resulted form Poisson's relation.

It is clear that the concerned distribution proves the random rule. But it has more tendencies to a kind of arrangement in relation to random rule.

2.5. Table of divisors of Burkhard

We express a skillful work from J.C.Burkhard in 1817 (divisors table), reprinted with permission from Gs. Carr from the famous book "Formulas and theorems in mathematics" (New York: Chelsea publishing company 1970) that it develops to "99000". This table calculates the least divisor of every integer number from "1" to "90000" that is not multiple of 2 or 3 or 5 (except prime numbers "P" which put O (Zero) against them).

A skillful work form J.C.Burkhard in 1817 that reprinted by printed by Permission from Gs. Carr from the famous book "Formulas and Theorems in Mathematics" (New York: Chelsea publishing company 1970) you can see a part of this table in the next page.

Table (V)

	00	03	06	09	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60	63	66	69	72	75	78	81	84	87
01	0	7	0	17	0	19	0	11	7	37	0	0	13	47	0	7	0	0	11	0	17	0	7	67	19	13	29	0	31	7
07	0	0	0	0	17	11	13	7	29	0	31	0	0	0	7	0	11	0	0	13	0	7	0	0	0	0	37	11	7	0
11	0	0	13	0	7	0	0	0	0	0	0	7	23	0	0	13	17	19	7	0	6	0	11	0	0	7	73	0	13	31
13	0	0	0	11	0	17	7	0	19	0	23	0	0	7	11	0	0	0	0	29	7	59	17	31	0	11	13	7	47	0
17	0	0	0	7	0	37	23	29	0	11	7	31	0	0	0	0	0	7	0	0	11	0	13	0	7	0	0	0	19	23
19	0	11	0	0	23	7	17	13	41	0	0	0	7	0	0	0	61	0	0	7	13	71	0	11	0	73	7	23	0	0
23	0	17	7	13	0	0	0	11	0	7	0	0	0	0	41	0	7	47	11	59	19	0	37	7	31	0	0	0	0	11
29	0	7	17	0	0	11	31	0	7	0	13	0	19	0	0	7	11	23	61	17	0	0	7	13	0	0	0	11	0	7
31	0	0	0	7	0	0	0	0	11	0	7	0	0	0	0	23	0	7	0	11	37	13	19	29	7	17	41	47	0	0
37	0	0	7	0	0	29	11	0	0	7	0	47	0	31	19	13	7	11	0	0	0	0	0	7	0	0	17	79	11	0
41	0	11	0	0	17	23	7	0	0	0	0	13	11	7	0	19	47	53	0	0	7	17	29	11	13	0	0	7	23	0
43	0	7	0	23	11	0	19	0	7	13	17	0	0	0	0	7	29	37	0	0	0	0	7	53	0	19	11	17	0	7
47	0	0	0	0	29	7	0	19	0	41	11	0	7	0	31	0	37	0	13	7	0	11	17	0	0	0	7	0	0	0
49	7	0	11	13	0	0	43	7	31	0	0	17	41	11	7	0	13	19	0	0	23	7	61	0	11	0	47	29	7	13
53	0	0	0	0	7	0	17	0	11	0	43	7	13	59	0	29	23	0	7	11	0	0	0	17	0	7	0	31	79	0
59	0	0	0	7	0	0	11	17	0	31	7	0	0	37	0	47	43	7	53	13	73	0	0	0	7	0	29	41	11	19
61	0	19	0	31	13	7	0	0	23	11	0	0	7	17	0	0	0	13	43	7	11	0	0	0	53	0	7	0	0	0
67	0	0	23	0	7	0	0	11	0	0	0	7	19	0	17	0	31	0	7	73	0	0	59	0	13	7	0	0	0	11
71	0	7	11	0	31	0	0	13	7	17	37	0	0	11	0	7	0	0	0	29	13	23	7	0	11	67	17	0	43	7
73	0	0	0	7	19	11	0	41	0	47	7	0	0	29	0	17	11	7	13	23	0	0	0	19	7	0	0	11	37	31
77	7	13	0	0	0	19	0	7	0	0	17	11	0	41	7	23	0	31	0	53	59	7	11	0	19	0	0	13	7	67
79	0	0	7	11	0	0	0	0	37	7	0	31	13	23	11	19	7	0	0	0	0	0	0	7	29	11	0	0	61	0
83	0	0	0	0	0	0	7	37	13	11	0	17	29	7	0	0	19	71	0	0	7	13	41	0	0	0	0	7	17	0
89	0	0	13	23	0	7	0	11	19	0	0	0	7	0	0	13	0	0	11	7	0	0	0	29	37	0	7	19	13	11
91	7	17	0	0	0	37	31	7	47	0	11	0	0	13	7	0	67	29	17	0	0	7	0	0	23	0	13	0	7	59
97	0	0	17	0	0	0	7	13	11	0	19	43	0	7	0	0	59	0	23	11	7	0	37	0	0	71	53	7	29	19

	01	04	07	10	13	16	19	22	25	28	31	34	37	40	43	46	49	52	55	58	61	64	67	70	73	76	79	82	85	88
01	0	0	0	7	0	0	0	31	41	0	7	19	0	0	11	43	13	7	0	0	0	37	0	0	7	11	0	59	0	13
03	0	13	19	17	0	7	11	0	0	0	29	41	7	0	13	0	0	11	0	7	17	19	0	47	67	0	7	13	11	0
07	0	11	7	19	0	0	0	0	23	7	13	0	11	0	59	17	7	41	0	0	31	43	19	7	0	0	0	29	47	0
09	0	0	0	0	7	0	23	47	13	63	0	7	0	19	31	11	0	0	7	37	41	13	0	43	0	7	11	0	67	23
13	0	7	23	0	13	0	0	0	7	29	11	0	47	0	19	7	17	13	37	0	0	11	7	0	71	23	41	43	0	7
19	7	0	0	0	0	0	19	7	11	0	0	13	0	0	7	31	0	17	0	11	29	7	0	0	13	19	0	0	7	0
21	11	0	7	0	0	0	17	0	0	7	0	11	61	0	29	0	7	23	0	0	0	0	11	7	0	0	89	0	0	0
27	0	7	0	13	0	0	41	17	7	11	53	23	0	0	0	7	13	0	0	0	11	0	7	0	17	29	0	19	0	7
31	0	0	17	0	11	7	0	23	0	19	31	47	7	29	61	11	0	0	0	7	0	59	53	79	0	13	7	0	19	0
33	7	0	0	0	31	23	0	7	17	0	13	0	0	27	7	41	0	0	11	19	0	7	0	13	0	17	0	0	7	11
37	0	19	11	17	7	0	13	0	43	0	0	7	37	11	0	0	0	0	7	13	17	41	0	31	11	7	0	0	0	0
39	0	0	0	0	13	11	7	0	0	17	43	19	0	7	0	0	11	13	29	0	7	47	23	0	41	0	17	7	0	0
43	11	0	0	7	17	31	29	0	0	0	7	11	19	13	43	0	0	7	23	0	0	17	11	0	7	0	13	0	0	37
49	0	0	7	0	19	17	0	13	0	7	47	0	23	0	0	0	7	29	31	0	11	0	17	7	0	0	0	73	83	0
51	0	11	0	0	7	13	0	0	0	0	23	7	11	0	19	0	0	59	7	0	0	0	43	11	0	7	0	37	17	63
57	0	0	0	7	23	0	19	37	0	0	7	0	13	0	0	0	0	7	0	0	47	11	29	0	7	13	72	22	43	17
61	7	0	0	0	0	11	37	7	13	0	29	0	0	31	7	59	11	0	67	0	61	7	0	23	17	47	19	11	7	0
63	0	0	7	0	29	0	13	31	11	7	0	0	53	17	0	0	7	19	0	11	0	23	0	7	27	79	0	0	0	0
67	0	0	13	11	0	0	7	0	17	47	0	0	0	7	11	13	0	23	19	0	7	29	67	37	62	11	31	7	13	0
69	13	7	0	0	37	0	11	0	7	19	0	0	0	13	17	7	0	11	0	0	31	0	7	0	0	0	12	0	11	7
73	0	11	0	29	0	7	0	0	31	13	19	23	7	0	0	0	0	0	0	7	0	0	13	11	73	0	7	0	0	19
79	0	0	19	13	7	23	0	43	0	0	11	7	0	0	29	0	13	0	7	0	37	11	0	0	47	7	79	17	23	13
81	0	13	11	23	0	41	7	0	29	43	0	59	19	7	13	31	17	0	0	0	7	0	0	73	11	0	23	7	0	83
87	11	0	0	0	19	7	0	0	13	0	0	11	7	61	41	43	0	17	37	7	23	13	11	19	83	0	7	0	31	0
91	0	0	7	0	13	19	11	29	0	7	0	0	17	0	0	0	7	11	0	43	41	0	0	7	19	0	61	0	11	17
93	0	17	13	0	7	0	0	0	0	11	31	7	0	0	23	13	0	67	7	71	11	43	0	41	0	7	0	0	13	0
97	0	7	0	0	11	0	0	0	7	0	23	13	0	17	0	7	19	0	29	0	0	73	7	47	13	43	11	0	0	7
99	0	0	17	7	0	0	0	11	23	13	7	0	29	0	53	37	0	7	11	17	0	67	13	31	7	0	19	43	0	11

	02	05	08	11	14	17	20	23	26	29	32	35	38	41	44	47	50	53	56	59	62	65	68	71	74	77	80	83	86	89
03	7	0	11	0	23	13	0	7	19	0	0	31	0	11	7	0	0	0	13	0	0	7	0	0	11	0	53	19	7	29
09	11	0	0	0	0	0	7	0	0	0	0	11	13	7	0	17	0	0	17	19	7	23	11	0	31	13	0	7	0	59
11	0	7	0	11	17	29	0	0	7	41	13	0	37	0	11	7	0	47	31	23	0	17	7	13	0	11	0	0	79	7
17	7	11	19	0	13	17	0	7	0	0	0	0	11	23	7	53	29	13	41	61	0	7	17	11	0	0	0	0	7	37
21	13	0	0	19	7	0	43	11	0	23	0	7	0	13	0	0	0	17	7	31	0	0	19	0	41	7	13	53	37	11
23	0	0	0	0	0	0	7	23	43	37	11	13	0	7	0	0	0	0	0	0	7	11	0	17	13	0	71	7	0	0
27	0	17	0	7	0	11	0	13	37	0	7	0	43	0	19	29	11	7	17	0	13	61	0	0	7	0	23	11	0	79
29	0	23	0	0	0	7	0	17	11	29	0	0	7	0	43	0	47	73	13	7	0	0	0	0	17	59	7	0	0	0
33	0	13	7	11	0	0	19	0	0	7	53	0	0	0	11	0	7	0	43	17	23	47	0	7	0	11	29	13	89	0
39	0	7	0	17	0	37	0	0	7	0	41	0	11	0	23	7	0	19	0	0	17	13	7	11	43	71	0	31	53	7
41	0	0	29	7	11	0	13	0	19	17	7	0	23	41	0	11	71	7	0	13	79	31	0	37	7	0	11	19	0	0

	02	05	08	11	14	17	20	23	26	29	32	35	38	41	44	47	50	53	56	59	62	65	68	71	74	77	80	83	86	89
47	13	0	7	31	0	0	23	0	0	7	17	0	0	11	0	47	7	0	0	19	0	0	41	7	11	61	13	17	0	23
51	0	19	23	0	0	17	7	0	11	13	0	53	0	7	0	0	0	0	0	11	7	0	13	0	0	23	83	7	41	0
53	11	7	0	0	0	0	0	13	7	0	0	11	0	0	61	7	31	53	0	0	13	0	7	23	29	0	0	0	17	7
57	0	0	0	13	31	7	11	0	0	0	0	0	7	0	0	67	13	11	0	7	0	79	0	17	0	0	7	61	11	13
59	7	13	0	19	0	0	29	7	0	11	0	0	17	0	7	0	0	23	0	59	11	7	19	0	0	0	0	13	7	17
63	0	0	0	0	7	41	0	17	0	0	13	7	0	23	0	11	61	31	7	67	0	0	0	13	17	7	11	0	0	0
69	0	0	11	7	13	29	0	23	17	0	7	43	53	11	41	19	37	7	0	47	0	0	0	67	7	17	0	0	0	0
71	0	0	13	0	0	7	19	0	0	0	0	0	7	43	17	13	11	41	63	7	0	0	0	71	31	19	7	11	13	0
77	0	0	0	11	7	0	31	0	0	13	29	7	0	0	11	17	0	19	7	43	0	0	13	0	0	7	41	0	0	47
81	0	7	0	0	0	13	0	0	7	11	17	0	0	37	0	7	0	0	13	0	11	0	7	43	0	31	0	17	0	7
83	0	11	0	7	0	0	0	0	0	19	7	0	11	47	0	0	13	7	0	31	61	29	0	11	7	43	59	83	19	13
87	7	0	0	0	0	0	0	7	0	29	19	17	13	53	7	0	0	0	11	0	0	7	71	0	0	13	0	0	7	11
89	17	19	7	29	0	0	0	0	0	7	11	37	0	59	67	0	7	17	0	53	19	11	83	7	0	0	0	0	0	89
93	0	0	19	0	0	11	7	0	0	41	37	0	17	7	0	0	11	0	0	13	7	19	61	0	59	0	0	7	0	17
99	13	0	29	11	0	7	0	0	0	0	0	59	7	3	11	0	0	0	41	7	0	0	0	23	0	11	7	37	0	0

This table calculates the least divisor of every integer number from 1 to 90000 that is not multiple of 2, 3 or 5 (except prime number "p" that put 0 in front of them). The first two digits of integer number (from left to right) are in row numbers from bold kind and the last two digits are in left the column. For example:

$$30012 = 4 \times 7503 = 3 \times 4 \times 2501 = 3 \times 4 \times 41 \times 61$$

Here, we ignore presenting another table for definition and identification prime divisors of numbers, because every one of these tables needs a long and extended calculations so that it may take a long time. It is obvious that none of these tables has special rule and order and we present the tables that solve the problem of composite numbers table and identification of prime numbers.